Final exam in Victoria, Australia

In the context of a pilot project, the Victorian Curriculum and Assessment Authority developed a different examination which allowed CAS. This was in addition to the existing examinations which assumed access to graphing calculator, which continued to be developed in their own right for that accredited study. These exams are for Year 12, which is the final year of secondary school. The Mathematical Methods(CAS) were the first 'CAS-assumed' exams in this state of Australia. The students did two exams (one was a multiple choice and short answer exam and the second one was the analysis task).75 students were involved in a pilot study where students were allowed to use CAS in their examinations.

In the following a comparison is made between the Non-CAS and the CAS-Assessment (Written Examination 2 – Analysis task). In 2003 more schools will be allowed to offer the new subject where CAS can be used in the examinations and the project group members are very interested to see how these CAS-exams evolve over the next couple of years

The CAS examinations (sample papers, supplementary questions, solutions, comments and advice in 2001, and actual examinations in 2003) were newly developed for the pilot study, and contain both common material and distinctive material with respect to the usual Mathematical Methods examinations.

The VCAA website**, www.vcaa.vic.edu.au/vce/studies/MATHS/caspilot.htm**, contains a comprehensive dedicated section for the CAS pilot which includes sample teaching and learning tasks, discussion papers and the like as well as links to the sample examinations, 2002 *examinations* **www.vcaa.vic.edu.au/vce/Exams/Maths/MathsMethodsCAS.htm**, (and soon the 2003 examination): and the examiners reports for the 2002 examinations, **www.vcaa.vic.edu.au/VCE/Assessmnet/AssessReports/Maths/MathsMethodCAS.htm.**

More information together with a rich source of papers on the use of CAS in Mathematics Education can be found at **www.edfac.unimelb.edu.au/DSME/CAS-CAT.**

Many thanks to Lynda Ball, University of Melbourne, for her valuable help and support.

Victorian Certificate of Education 2002

Written Examination 2 (Analysis task)

See references at the end of the paper

An Assessment for two groups of students

Reading time 15 minutes
Working time 90 minutes

Non CAS Assessment

Students are permitted to bring .....
.... and an approved scientific and/or graphics calculator (memory may be retained).

Instructions

- Answer **all** questions in the space provided.
- A decimal approximation will be not accepted if an exact answer is required to a question.
- Where an exact answer is required to a question, appropriate working must be shown.
- Where an instruction to **use calculus** is stated for a question, you must show an appropriate derivative or antiderivative.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Total 55 marks

CAS-Assessment

Students are permitted to bring .....
.... and an approved scientific and/or computer algebra (CAS) calculator (memory may be retained).

Instructions

- Answer **all** questions in the space provided.
- A decimal approximation will be not accepted if an exact answer is required to a question.
- Appropriate work must be shown if more than one mark is available.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Total 55 marks

Question 1

A well-designed computer screen display aims to make it quick and easy for a user to do tasks such as clicking on a screen button. Fitt´s Law models the way in which the time taken to move to, and click on, a screen button depends on the distance the mouse is moved and the width of the screen button. According to Fitt´s Law, for a fixed distance travelled by the mouse, the time taken, in seconds, is given by $a - b \log_e(x)$, $0 < x \le 5$, where x cm is the button width and a and b are positive constants for a particular user.

a) Minnie discovers that, for her, $a = 1.1$ and $b = 0.5$

(i) Let $f: (0,5] \to R$, $f(x) = 1.1 - 0.5 \log_e(x)$.
Sketch the graph of $y = f(x)$ on the axes below. Label any asymptote with its equation and any end-point with its exact coordinates. *3 marks*

(ii) Explain why f^{-1}, the inverse function of f, exists. *1 mark*

(iii) Find $f^{-1}(x)$, the rule for f^{-1}. *2 marks*

(iv) State the domain of f^{-1}. *1 mark*

(v) Sketch the graph of $f^{-1}(x)$ on the axes below. Label any asymptote with its equation and any end-point with its exact coordinates. *2 marks*

From VCE Mathematical Methods (CAS) Pilot Study Examination 2 (abbreviated)

b) Mickey decides to find values of a and b for his use. He finds that when $x = 1$, his time is 0.5 seconds, and when x is 1.5, his time is 0.3 seconds.

Non-CAS-Assessment

c) Show that, when the button width is halved, the time taken by Minnie (for whom $a = 1.1$ and $b = 0.5$) is increased by $\log_e \sqrt{2}$ seconds.

3 marks

Question 1 *Total 14 marks*

CAS-Assessment

c) Solve the equation
$k\,(1.1 - 0.5 \log_e(x)) = T$
for x, where k and T are positive real numbers.

1 mark

c) Let g: $[0,5] \rightarrow R$, $g(x) = k\,(1.1 - 0.5 \log_e(x))$, where k is positive and real. Given a positive real number T, find the largest value of k such that the equation $g(x) = T$, has a solution for x in the domain of g.

2 marks

Question 1 *Total 14 marks*

Question 2

Emmy is gathering data on two particular species of yellow butterflies, Fhaisi and Jojo, which are very difficult to tell apart. Both species are equally likely to be caught.

One technique for telling the difference between the two species is by measuring the length of their antennas. For Fhaisi butterflies, antenna lengths are normally distributed with a mean of 20 mm and a standard deviation of 2 mm.

a) Find the probability, correct to three decimal places, that a randomly selected Fhaisi butterfly antenna is shorter than 16 mm. *2 marks*

b) 8% of Jojo butterfly antennas are shorter than 19 mm and 8% of Jojo butterfly antennas are longer than 28 mm. Assume that the antenna length of Jojo butterfly antennas are also normally distributed. Find the mean and the standard deviation of antenna length of Jojo butterflies, to the nearest 0.1 mm. *4 marks*

c) In the region where Emmy is butterfly hunting, 20% of the yellow butterflies are Jojos and the other 80% are Fhaisis. The probability that a randomly selected Jojo butterfly antenna is shorter than 20 mm is 0.1370.

(i) Calculate the proportion, correct to three decimal places, of **all** Jojo and Fhaisi butterfly antennas that are shorter than 20 mm. *2 marks*

(ii) Emmy examined a single butterfly antenna. It was shorter than 20 mm.
What is the probability, correct to three decimal places, that it is a Fhaisi antenna? *2 marks*

Non-CAS-Assessment

(iii) Find the probability, correct to three decimal places, that a random sample of 10 yellow butterflies from this region will contain exactly 4 Jojo butterflies.

2 marks

CAS-Assessment

d) Let X be the random variable with values equal to the distance in metres of a Fhaisi butterfly from an old tree.

The probability density function of X is

$$f(X)=\begin{cases}\dfrac{2x}{a^2} & 0\le x\le a, \quad a \text{ is a constant}\\ 0 & \text{otherwise}\end{cases}$$

(i) It is found that the mean distance of a butterfly from the old tree is 150m. Show that the value of a is 225.

2 marks

(ii) Find the probability, correct to three decimal places, of a Fhaisi butterfly being within 200 m of the old tree.

2 marks

Question 2 *Total 12 marks*

Question 2 *Total 14 marks*

Non-CAS-Assessment

Question 3

a) Write down an equation in x, the solutions of which give the x-coordinates of the stationary points of the curve whose equation is

$$y=\frac{1}{2}(2x^4-x^3-5x^2+3x).$$

2 marks

CAS-Assessment

Question 3

a) (i) The polynomial $2x^4-x^3-5x^2+3$ can be factorised as $x(2x-3)(ax2+bx+c)$.

Find the values of a, b and c.

1 mark

(ii) Find the exact value solutions of the equation

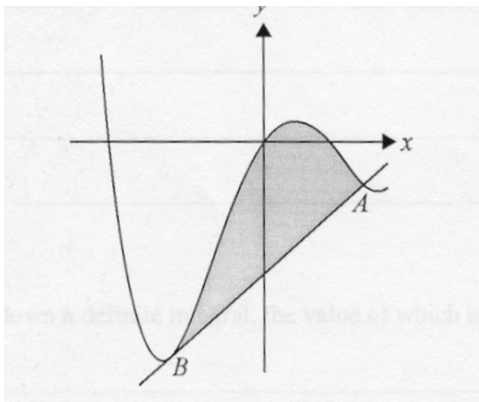
$$2x^4-x^3-5x^2+3x=0.$$

2 marks

b) The diagram shows the curve whose equation is $y=\frac{1}{2}(2x^4-x^3-5x^2+3x)$ and the normal to the curve at A, where $x = 1$.

(i) Show that the equation of this normal is $y = x - 1.5$.

3 marks



From VCE Mathematical Methods (CAS) Pilot Study Examination 2 (abbreviated)

(ii) Show that this normal is a tangent to the curve at a point *B* and find the exact values of the coordinates of *B*. *4 marks*

c) (i) Write down a definite integral, the value of which is the area of the shaded region. *2 marks*

(ii) Find the area of the shaded region, correct to two decimal places. *1 mark*

Non-CAS-Assessment		**CAS-Assessment**	
Question 3	*Total 12 marks*	*Question 3*	*Total 13 marks*

Question 4

On an adventure park ride, riders are strapped into seats on a platform which starts 15 metres above the ground and goes up and down. The distance, *x* metres, of the platform above the ground, *t* seconds after the ride starts can be modelled by the formula $x(t) = 15 + 6\sin\left(\frac{\pi t}{3}\right)$.

a) (i) According to this model, find the maximum height above the ground reached by the platform. *1 mark*

Non-CAS-Assessment

(ii) According to this model, how many seconds after the ride starts is the platform first closest to the ground and how high above the ground is it at that time? *2 marks*

CAS-Assessment

(ii) According to this model, how many seconds after the ride starts is the platform first exactly 9 metres from the ground? *1 mark*

Tasmania Jones is redesigning the ride so that the platform moves further up and down each cycle. During the first 60 seconds of the redesigned ride, the distance, *y* metres, of the platform above the ground, *t* seconds after the ride starts, con be modelled by the formula

$$y(t) = 15 + e^{0.04t}\sin\left(\frac{\pi t}{3}\right),\ 0 \le t \le 60.$$

b) (i) According to this model the platform is exactly 6 metres above the ground for the first time about 58 seconds into the ride. Find this time correct to two decimal places of a second. *1 mark*

(ii) According to this model, how many times is the platform exactly 15 metres above the ground from t = 40 to t = 59? *1 mark*

(iii) According to this model, find the time which passes from when the ride starts until the platform first reaches 24 metres above the ground. Give your answer correct to the nearest second. *2 marks*

From VCE Mathematical Methods (CAS) Pilot Study Examination 2 (abbreviated)

c) (i) Find an expression for $\frac{dy}{dt}$. *2 marks*

(ii) Hence write down an equation, one solution of which is the value of t, when the platform is closest to the ground. Find this value of t, correct to two decimal places. Also find, according to the model, the distance of the platform above the ground at that time, correct to two decimal places of a metre. *3 marks*

d) Tasmania can adjust the ride so the model for the distance, in metres, of the platform above the ground *t* seconds after the ride starts becomes

$h(t) = 15 + a\,e^{0.04t}\sin\left(\frac{\pi t}{3}\right)$, $0 \le t \le 60$, where *a* is a positive constant.

Find, correct to three decimal places, the greatest value of *a* such that *h*' is never more than 11 during the first 60 seconds of the ride.

3 marks

Non-CAS-Assessment

e) Safety regulations for the ride require that

$-11 \le \frac{dy}{dt} \le 11$ during the first 60 seconds.

Find the range of values of *t*, correct to three decimal places, for which $-11 \le \frac{dy}{dt} \le 11$ during the first 60 seconds of the ride.

2 marks

Question 4 *Total 17 marks*

CAS-Assessment

Question 4 *Total 14 marks*

References

Victorian Curriculum and Assessment Authority (2002). *VCE Mathematical Methods (CAS) Pilot study: Written examination 2*. Melbourne: Author.

Victorian Curriculum and Assessment Authority (2002). *VCE Mathematical Methods (CAS) Pilot study: Written examination 1*. Melbourne: Author.