9. Plenary demonstrations

introduction

Introduction

For a mathematics teacher there are several ways to use TI Interactive!:

- as a "dynamic" tool, to prepare lessons at home, or for your students to perform well adapted exercises in the class or at home,
- as a "super blackboard" to be handled by the teacher in front of the class to illustrate small lesson sequences in which the interactivity of the software provides additional value compared to normal blackboard and chalk.

The subjects treated in the following files are supposed to fit into the last category.

content

Content

Demonstrations for two topics are offered:

9.1 The slope as a characteristic property of the line	Grades 9 or 10
9.2 Matrices and beetles	Grade 11

procedure

Working procedure

For both topics there is a teacher guide file and one or more demonstration files. You can open them by clicking on the blue hyperlinks below. Each of these files contains links to the other files of the group. Once different files are open, do not use the links between them any more, simply switch from one to another using the buttons on the Status Bar.

If you want to keep the initial content of the present files unchanged, before beginning to work you must eithe*r* put the files into "read only" mode so that it is not possible to erase or change parts by mistake (in Windows Explorer, right-click on the name of the file, choose Properties and select read only) or save the files under a different name.

9.1 The slope, a characteristic property of the line

introduction

Introduction

The aim of this demo is to illustrate graphically the fact that the slope is a characteristic property of the line, in the sense that if you consider the quotient of the difference of the ordinates and of the difference of the abscissas of any two distinct points of a curve, this quotient is a constant if and only if the curve is a straight line non parallel to the *Oy* axis.

instructions

Instructions

The slope demo file contains 2 pages separated by a Math Section Break.

On the first page, you should define a function $\mathrm{f1}(x) = m \cdot x + 1$ where *m* is a variable parameter controlled with a slider bar (we call SBm).

You should also define a fixed reference point A (square, blue colored) with coordinates (a, $\mathrm{f1}(a)$) and a variable point B (round and red) with coordinates (b, $\mathrm{f1}(b)$) where *b* is controlled by a second slider bar (we call SBb).

The left graph shows the two points A and B on the graph of the function $\mathrm{f1}(x)$ as well as the segments representing the differences of their abscissas and of their ordinates (*).

The right graph shows a round and green point W with coordinates (b, $\mathrm{w1}(b)$) where

$\mathrm{w1}(b) = \dfrac{\mathrm{f1}(b) - \mathrm{f1}(a)}{b - a}$ is the slope calculated with respect to the actual points A and B.

a. *Leave SBm and the variable a unchanged and animate SBb.*

On the left graph : Point B moves on the graph of $\mathrm{f1}(x)$ and the construction (*) follows.
On the right graph : Point W moves on a horizontal line which means that $\mathrm{w1}(b)$ is a constant.

b. *Using SBm, change the value of m while leaving the variable a unchanged. Animate SBb again.*

This results in another straight line with a constant slope that has a value equal to the value of *m*.

c. *Repeat the previous question for a different value of a.*

This affects the choice of the fixed reference point but the result is always the same.

On the second page, you find exactly the same scenario but with a function $\mathrm{f1}(x)$ that you choose and define. For this example choose a different polynomial function which is not of degree 1.

It is easy to see that the value of $\mathrm{w1}(b) = \dfrac{\mathrm{f1}(b) - \mathrm{f1}(a)}{b - a}$ is no longer a constant!

<u>Remark</u> : the graph of $\mathrm{w1}(b)$ (green dotted line) is NOT the graph of the derivative of function $\mathrm{f1}(x)$!

demo

Demo 1

The value of m

define $\mathrm{f1}(x) = m \cdot x + 2$

$a := 1$

The value of b

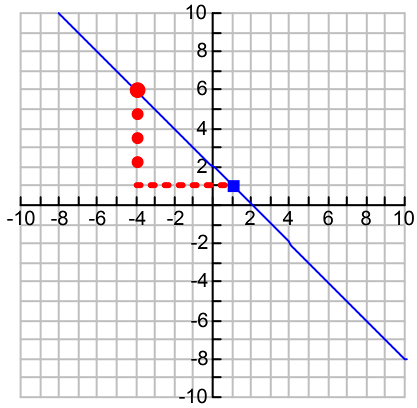
Graph of $\mathrm{f1}(x)$: blue graph

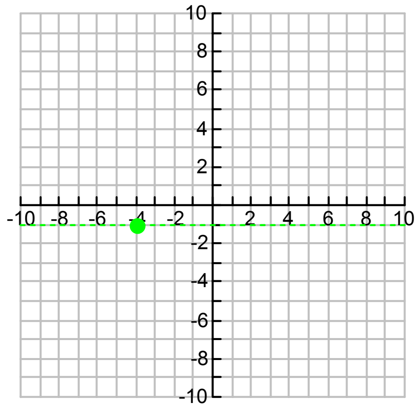
Blue square point: reference point $(a, \mathrm{f1}(a))$

Thick red round point: $(b, \mathrm{f1}(b))$

$$\mathrm{w1}(b) = \frac{\mathrm{f1}(b) - \mathrm{f1}(a)}{b - a}$$

Green round point: $(b, \mathrm{w1}(b))$





demo

Demo 2

define $\mathrm{f1}(x) = x^2$

$a := 1$

The value of b

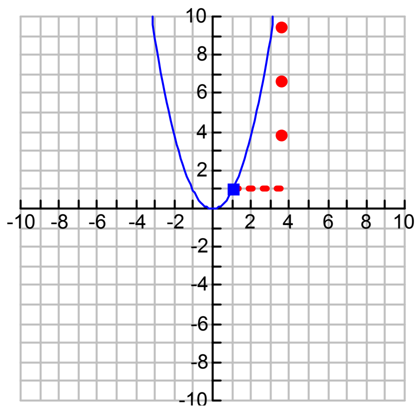
Graph of $\mathrm{f1}(x)$: blue graph

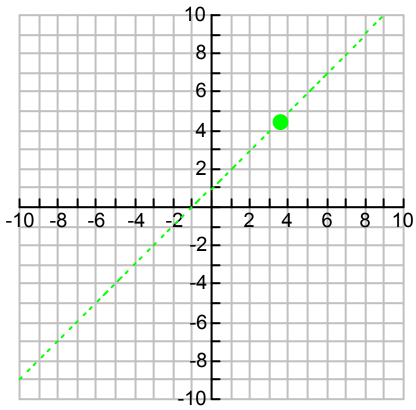
Blue square point: reference point $(a, \mathrm{f1}(a))$

Red round point: $(b, \mathrm{f1}(b))$

$$\mathrm{w1}(b) = \frac{\mathrm{f1}(b) - \mathrm{f1}(a)}{b - a}$$

Green round point: $(b, \mathrm{w1}(b))$





9.2 Matrices and beetles

introduction

Introduction

The starting point of this demo is an application task on matrices and matrix operations. It consists of an exercise relative to a population of insects.

The demo contains several steps, taking into account the interactivity within a TII document and the possibility to introduce sliders.

A possible scenario is described in the instructions.

starting task

Starting Task

A population of beetles is developing according to the following rules.

- None of the beetles reaches the age of 3 years. During their first year, only one quarter of the beetles survive. Among those that survive, only 50% are alive at the end of the second year.
- Let us call adults (A) the beetles that reach the age of two, young (Y) the beetles that are one year old and new (N) the new born beetles.
- The number of males is always equal to the number of females.
- Each young female beetle (Y) can have four baby beetles, each adult female beetle (A) can have eight babies.
- Initially the population of beetles is composed of 200 adults, 400 young and 800 new.

Find the matrix M which enables you to determine the population in a given year if the population in the preceding year is known. Use M to calculate the number of new, young and adult beetles after two years.

instructions

Instructions

We recommend the following scenario for demonstration.

a. Ask the students to write down the answers to the exercise on paper.

b. Use the file M&B1 to explain and show the correct answer.

c. Use the interactivity within a TII document to investigate what happens to the population if we look further than two years ahead. Change the value of t (take t equal to 20) and observe the result (also in M&B2).

d. Consider a new question:

How does the total population of beetles develop during a period of 20 years?

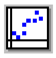
To investigate this, the first idea is to calculate the sum N + Y + A when t equals 20 (see M&B3).

It is clear that the total number of beetles is bigger than in the beginning (initially the total number of insects was 1400) but the way the population is developing is not so clear ... unless we make a graph of the total number of beetles in function of t ! (see M&B4).

e. Make use, once more, of the interactivity of TII and ask a new question:

What is the incidence on this population of the use of pesticides whose effect is to divide by two the birth rate of the beetles?

To investigate this, it is sufficient to change the values of some elements of the matrix M of the file M&B4, as has been done in file M&B5.

Hint : if you change elements of matrix M in file M&B4, it will be necessary to click on  in the Graph window, in order to have the graph updated.

f. Let us go still further and use the fact that TII provides the possibility to insert sliders ...

Suggest a last exercise:

Suppose that the impact of pesticides on the rate of birth rate is sufficiently well known to be able to predict that each female beetle (Y) will have 4a baby beetles and each female beetle (A) 8a babies with parameter a varying between 1.02 and 0.98 according to the quantity of pesticides used.

Study the influence of a (and hence of the quantity of pesticides) on the growth of the population.

Take file M&B4, insert a slider correponding to a parameter *a* whose values range from 0.98 to 1.02 and change the elements of the first row of M into 0, 2*a*, 4*a* or load file M&B6.
Observe the effect on the population of beetles when scrolling the slider button.

demo

Demo 1

Taking into account that the number of females is equal to the half of the total number of beetles, after one year, the number of new beetles (*N*) is given by:

$$N = 4\cdot\left(\frac{400}{2}\right)+8\cdot\left(\frac{200}{2}\right)=2*400+4*800$$

The number of young beetles (*Y*) is given by: $Y=\left(\frac{1}{4}\right)\cdot 800$

The number of adult beetles (*A*) is given by: $A=\left(\frac{1}{2}\right)\cdot 400$

Let us call ***P* the initial population vector**: $P := \begin{bmatrix}800\\400\\200\end{bmatrix}$

The **population growth matrix *M*** is then: $M := \begin{bmatrix}0 & 2 & 4\\ \frac{1}{4} & 0 & 0\\ 0 & \frac{1}{2} & 0\end{bmatrix}$

And the **population vector after *t* years** is given by ***R*(*t*)** with $R(t) := M^t\cdot P$.

So, when $t := 2$: $R(t) \quad \begin{bmatrix}1200\\400\\100\end{bmatrix}$

Demo 2

Let us call ***P*** **the initial population vector**: $P := \begin{bmatrix} 800 \\ 400 \\ 200 \end{bmatrix}$

The **population growth matrix** ***M*** is then: $M := \begin{bmatrix} 0 & 2 & 4 \\ \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \end{bmatrix}$

And the **population vector after** ***t*** **years** is given by ***R*(*t*)** with $R(t) := M^t \cdot P$.

So, when $t := 20$:

$$R(t) \quad \begin{bmatrix} \frac{40975}{32} \\ \frac{20475}{64} \\ \frac{20475}{128} \end{bmatrix} \quad \text{and} \quad \text{approx}(R(t)) \quad \begin{bmatrix} 1280. \\ 320. \\ 160. \end{bmatrix}$$

Demo 3

Let us call ***P*** **the initial population vector**: $P := \begin{bmatrix} 800 \\ 400 \\ 200 \end{bmatrix}$

The **population growth matrix** ***M*** is then: $M := \begin{bmatrix} 0 & 2 & 4 \\ \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \end{bmatrix}$

And the **population vector after** ***t*** **years** is given by ***R*(*t*)** with $R(t) := M^t \cdot P$.

So, when $t := 20$:

$$R(t) \quad \begin{bmatrix} \frac{40975}{32} \\ \frac{20475}{64} \\ \frac{20475}{128} \end{bmatrix} \quad \text{and} \quad \text{approx}(R(t)) \quad \begin{bmatrix} 1280. \\ 320. \\ 160. \end{bmatrix}$$

The **total population Tot(*t*) after *t* years** is the sum of the elements of $R(t)$.
This number is the last element of the cumulative sum of $R(t)$.

Let us define: $C(t) := \text{cumSum}(R(t))$

$\text{Tot}(t) := (C(t))_{[3,\,1]}$

Finally, the total number of beetles after t years is: $\text{approx}(\text{Tot}(t)) \quad 1760.$

demo

Demo 4

Let us call ***P*** **the initial population vector**: $P := \begin{bmatrix} 800 \\ 400 \\ 200 \end{bmatrix}$

The **population growth matrix *M*** is then: $M := \begin{bmatrix} 0 & 2 & 4 \\ \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \end{bmatrix}$

And the **population vector after *t* years** is given by ***R*(*t*)** with $R(t) := M^t \cdot P$.

So, when $t := 20$:

$$R(t) \quad \begin{bmatrix} \frac{40975}{32} \\ \frac{20475}{64} \\ \frac{20475}{128} \end{bmatrix} \quad \text{and} \quad \text{approx}(R(t)) \quad \begin{bmatrix} 1280. \\ 320. \\ 160. \end{bmatrix}$$

The **total population Tot(*t*) after *t* years** is the sum of the elements of $R(t)$.
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$$\text{Tot}(t) := (C(t))_{[3,\,1]}$$

Finally, the total number of beetles after *t* years is: $\text{approx}(\text{Tot}(t))$ 1760.

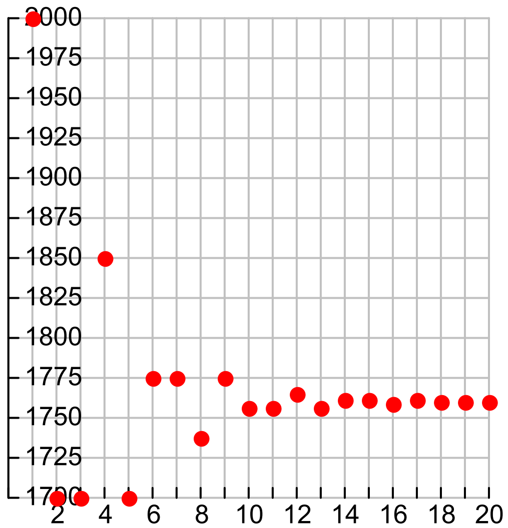
Graph of the total number of beetles Tot(*t*) in function of the year *t* $(1 \le t \le 20)$

Let us define

$u := \text{seq}(t, t, 1, 20, 1)$ $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$

$v := \text{seq}(\text{Tot}(t), t, 1, 20, 1)$

to obtain the following graph:



demo

Demo 5

Let us call ***P* the initial population vector**: $P := \begin{bmatrix} 800 \\ 400 \\ 200 \end{bmatrix}$

The **population growth matrix *M*** is then: $M := \begin{bmatrix} 0 & 1 & 2 \\ \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \end{bmatrix}$

And the **population vector after *t* years** is given by ***R*(*t*)** with $R(t) := M^t \cdot P$.

So, when $t := 20$:

$$R(t) \quad \begin{bmatrix} \frac{136575}{32768} \\ \frac{22425}{16384} \\ \frac{59075}{65536} \end{bmatrix} \quad \text{and} \quad \text{approx}(R(t)) \quad \begin{bmatrix} 4. \\ 1. \\ 1. \end{bmatrix}$$

The **total population Tot(*t*) after *t* years** is the sum of the elements of $R(t)$.

This number is the last element of the cumulative sum of $R(t)$.

Let us define: $C(t) := \text{cumSum}(R(t))$

$$\text{Tot}(t) := (C(t))_{[3,\,1]}$$

Finally, the **total number of beetles after *t* years** is: $\text{approx}(\text{Tot}(t)) \quad 6.$

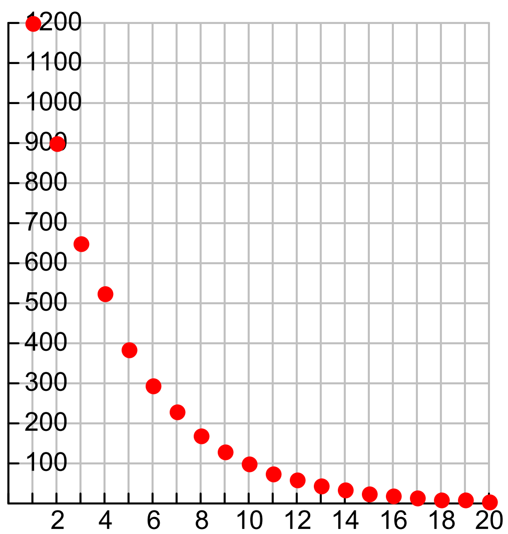
Graph of the total number of beetles Tot(*t*) in function of the year *t* $(1 \le t \le 20)$

Let us define

$u := \text{seq}(t, t, 1, 20, 1) \quad \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$

$v := \text{seq}(\text{Tot}(t), t, 1, 20, 1)$

to obtain the following graph:



Demo 6

a:=1.01

Let us call ***P*** **the initial population vector**: $P := \begin{bmatrix} 800 \\ 400 \\ 200 \end{bmatrix}$

Suppose the **population growth matrix** ***M*** is: $M := \begin{bmatrix} 0 & 2\cdot a & 4\cdot a \\ \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \end{bmatrix}$

and the **population vector after** ***t*** **years** is given by ***R(t)*** with $R(t) := M^t \cdot P$

So, when $t := 20$:

$$R(t) \quad \begin{bmatrix} \frac{178172270882201755492239}{128000000000000000000} \\ \frac{44339996305727797282851}{128000000000000000000} \\ \frac{176650229492529966707}{1024000000000000000} \end{bmatrix} \quad \text{and} \quad \text{approx}(R(t)) \quad \begin{bmatrix} 1392. \\ 346. \\ 173. \end{bmatrix}$$

The **total population Tot(*t*) after *t* years** is the sum of the elements of $R(t)$.

This number is the last element of the cumulative sum of $R(t)$.

Let us define : $C(t) := \text{cumSum}(R(t))$

$$\text{Tot}(t) := (C(t))_{[3,\,1]}$$

Finally, the **total number of beetles after *t* years** is: $\text{approx}(\text{Tot}(t)) \quad 1911.$

Graph of the total number of beetles Tot(*t*) in function of the year *t* $(1 \le t \le 20)$.

Let us define

$u := \text{seq}(t, t, 1, 20, 1)$

$v := \text{seq}(\text{Tot}(t), t, 1, 20, 1)$

to obtain the following graph: