

6. Periodic waves

Goal of the task

In physics students need some mathematical skills to understand the phenomenon of periodic waves. To understand waves is essential in physics, so the mathematical model for waves ought to be a central part of a study in physics. The goal of this task is to give students some associations between the mathematical model and the phenomenon in physics they are studying.

The main task is to study the mathematical model for the wave function and the effect on the graphical picture when we are changing the values for the parameters in the function. Using the hints should enable students to find the connection between the values of the parameters and the shape of the graph.

Target group and required time

The target group for this task are eleventh-grade mathematics and physics students (17 years old) who are unfamiliar with both the effect of changing parameter values in a parametrised function and mathematical modelling.

The task requires approximately 4 to 6 lessons in the computerlab with additional time at home. The work could contribute to the students' understanding of mechanical waves, particularly if done after an introduction into the topic.

Preliminary TI InterActive! skills

The students need some basic skills in using TII for graphing and for applying algebraic commands such as the solve command.

Preliminary mathematical skills

The students need some knowledge about graphing and about trigonometric functions.

File organization

The task consists of the following linked TII files:

- Notebook.tii: The file in which the student writes solutions and the starting point containing hyperlinks to the other files.
- MainTask.tii: The file that briefly describes the task.
- Hint1.tii, Hint2.tii, ..., Hint5.tii: Files that suggest a question to start with if the student doesn't know what to do with the task.
- Addition_Physics.tii: Contains an additional task which focuses on the relationships between the representations in mathematics and physics.
- Definition.tii: File containing some basic knowledge concerning periodic functions
- Meaning_phys.tii: File containing the different representations in mathematics and physics
- Solution.tii: Contains the solutions.

The hyperlinks only work if the files are installed in the map c:\TII\PeriodicWaves.

Technical hints

Students start by opening the notebook file in TII. It is recommended that the main task is opened in a separate window so that two versions of TII are activated. By means of a right mouse click in the Windows menu bar the option 'Windows Cascade' offers opportunities for switching easily between the files. However, if the same hyperlink is used twice two versions of the same file will be opened! This may be a source of confusion and therefore requires some attention by the teacher.

As TII only supports absolute hyperlinks and no relative links, the links need to be adjusted after installation of the files, for example on the school network.

Physical Meaning

description

In accordance with one of the theses in quantum mechanics, all particles in nature can be associated with a wave. Thus it is of great importance that we understand the mechanism which influences the movement of the waves.

With a PERIOD we mean a movement that repeats itself after a constant time lag T .

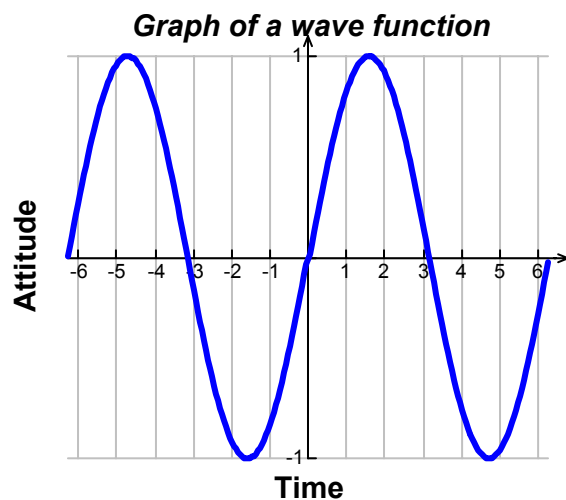
We say the movement is periodic with T as period.

The wave function f is given by
 $f(x) := A \cdot \sin(B \cdot x + C) + D$.

With given values:

$A := 1$, $B := 1$, $C := 0$ and $D := 0$.

We get the graph shown at the right.



This task is a study of wave functions, which are also called functions for harmonic waves. The investigation is connected to the vital importance that values of the parameters A , B , C and D have on the shape of the graph.

The additional task concerns phenomena in physics which are influenced by the parameter values in the general formula of the wave function.

period

Period: If any quantity is a function of the time, and this function repeats itself exactly after constant time intervals T , the quantity is said to be periodic, and T is called the period of the function.

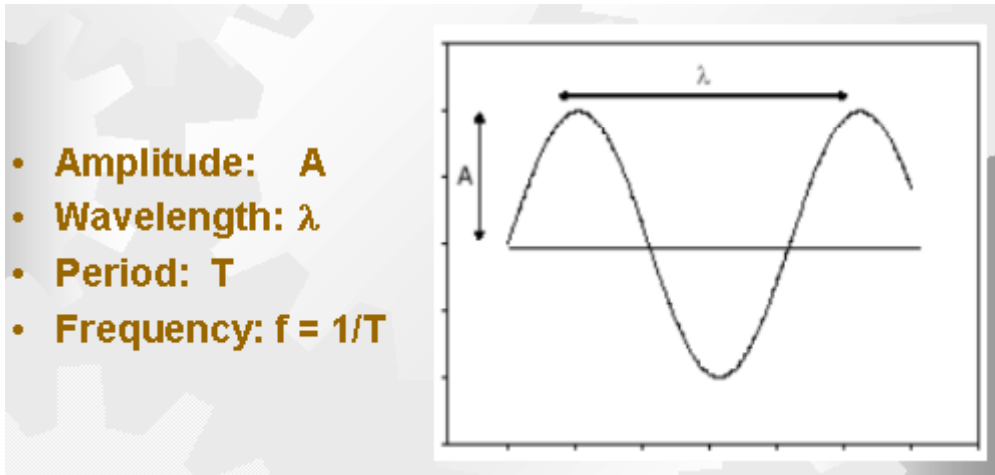
interval

Time interval T : The time a movement is using to swing a period.

wavelength

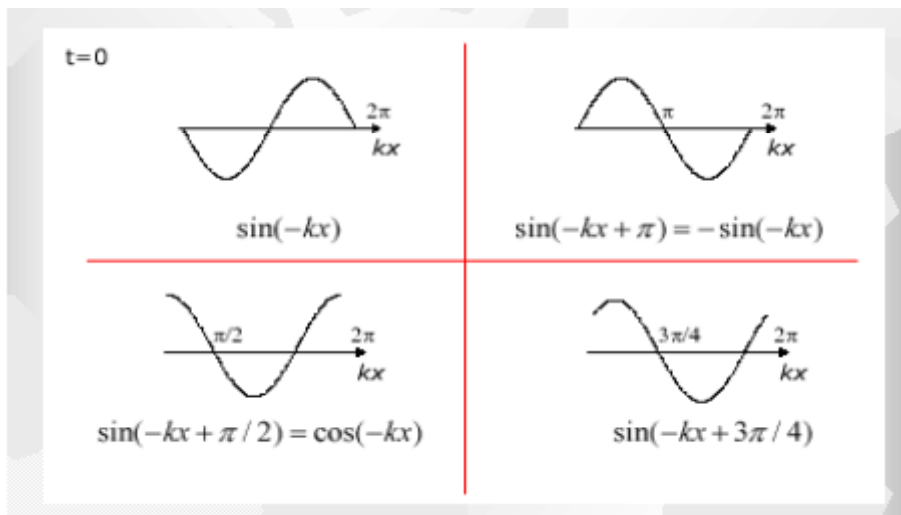
Wavelength λ : The distance between successive points of equal phase of a wave.

Phase: Points in the path of a wave motion are said to be points of equal phase if the displacements at those points at any instant are exactly similar; i.e. of the same magnitude and varying in the same manner.



The **velocity** of the wave: $v = \lambda \cdot f = \frac{\lambda}{T}$.

The following graphs illustrate the phase shift:



6.1 Main task

task

The general wave function f is given by $f(x) := A \cdot \sin(B \cdot x + C) + D$.

In what way is the graph of f influenced by the values each of the following parameters?

- a. Parameter A
- b. Parameter B
- c. Parameter C
- d. Parameter D

Hint 1

Hint 2

Hint 3

Hint 4

6.2 Additional physics task

intro

The wave function in mathematics is often represented as follows: $f(x) = A \cdot \sin(B \cdot x + C) + D$.

In physics, the following representation is most common: $f(x) = A \cdot \sin(k \cdot (x - v))$ with $v = \lambda \cdot f = \frac{\lambda}{T}$.

Here v is the wave velocity, λ is the wavelength, f is the frequency (and not the name of the function anymore!) and T is the period.

task

- a. Which parameter determines the amplitude?
Give an example which shows the physical effect.
- b. How do we decide on the wave length for the model?
Give an example which shows the physical effect.
- c. In what way is the graph of the wave function influenced by the wave's frequency and velocity?

Hint 5

6.3 Hints

hint 1

The value of the parameter A

Ensure that TI InterActive! is set to radians!

In what way is the graph influenced by the values of the parameter A ?

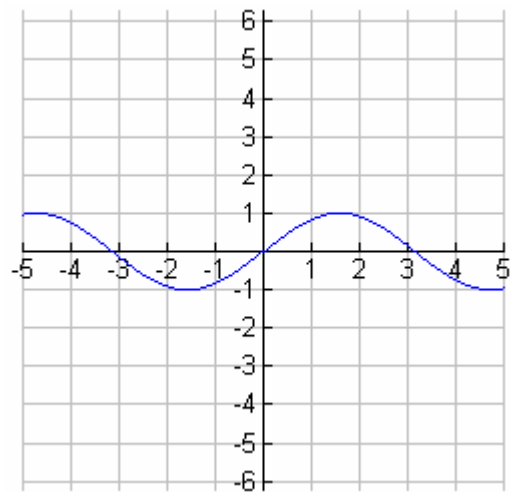
The function f is given by $f(x) := A \cdot \sin(B \cdot x + C) + D$.

Given values: $B := 1$, $C := 0$ and $D := 0$.

The value to A can be changed using the slider.



$$f(x) = \sin(x)$$



hint 2

The value of the parameter B

In what way is the graphical picture influenced by the values given to the parameter B ?

Ensure that TI InterActive! is set to radians!

We are using a reference curve $g(x) := \sin(x)$.

The function f is given by $f(x) := A \cdot \sin(B \cdot x + C) + D$.

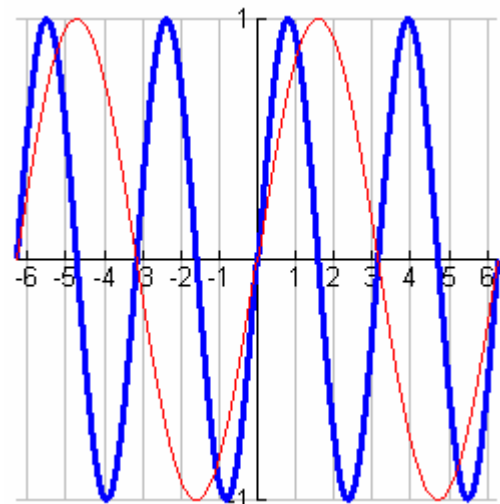
Given values: $A := 1$, $C := 0$ and $D := 0$.

The value of B can be changed using the slider.



$$f(x) = \sin(2x)$$

You can calculate the zeros with the solve command.



hint 3

The value of the parameter C

Ensure that TI InterActive! is set to radians!

In what way is the graph influenced by the values of the parameter C ?

We are using a reference curve: $g(x) := \sin(x)$.

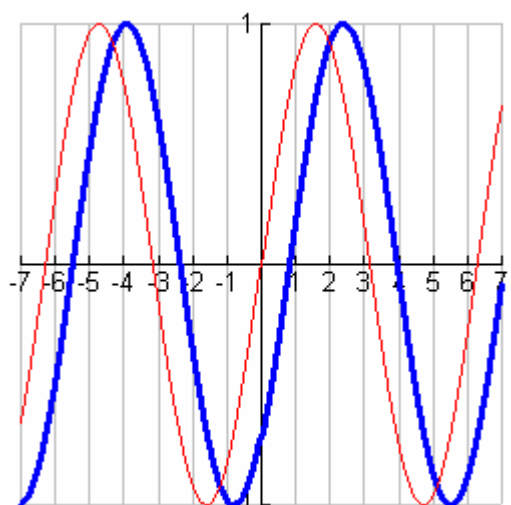
The function f is given by $f(x) := A \cdot \sin(B \cdot x + C) + D$.

Given values: $A := 1$, $B := 1$ and $D := 0$.

The value for the parameter C can be changed using the slider.



$$f(x) = \sin(x - .8)$$



The value of the parameter D

Ensure that TI InterActive! is set to radians!

In what way is the graphical picture influenced by the values given to the parameter D ?

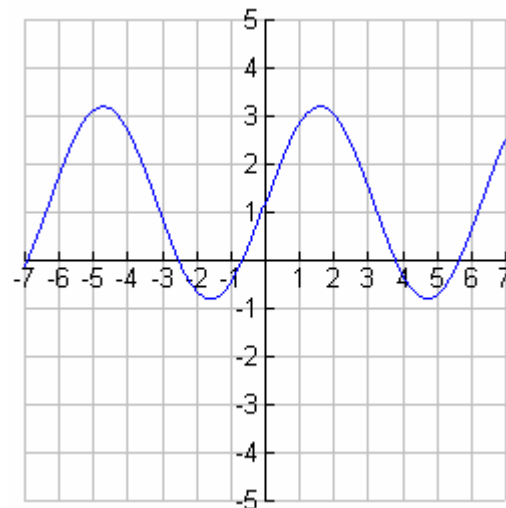
The function f is given by $f(x) := A \cdot \sin(B \cdot x + C) + D$.

Given values: $A := 2$, $B := 1$ and $C := 0$.

The value to D can be changed using a slider.

$D = 1.2$

$$f(x) = 2 \cdot \sin(x) + 1.2$$



The value of the parameter A

We graph a reference curve: $g(t) := A \cdot \sin(t)$.

Instead of changing the parameter's value directly, we can use a slider to study the dynamics.

The function f is given by $f(t) := A \cdot \sin(k \cdot t + C)$.

The value of parameter A :

$A = 2.5$

The value of parameter k :

$k = 2$

The value of parameter C :

$C = 1.4$

This is the graph of the function

$$f(t) = 2.5 \sin(2t + 1.4)$$

